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ABSTRACT

Experimental studies conducted since 1980 were reviewed. Each of the 34 studies concerning problem solving is briefly summarized, followed by comments, criticisms, and/or concerns. A summary indicates that experimental research of the 1980s seems to focus on: (1) children's processes, understanding, functioning, difficulties, and representations; (2) instructional models and techniques; and (3) the language, readability, structure, format, and syntax of problems. Omissions of topics, general findings, and criticisms are noted. Then 34 "major findings" are listed, followed by 55 suggestions and questions for further research. Forty-two references are provided, and an appendix presents three general semantic categories for addition and subtraction word problems. (MNS)

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WHAT IS EXPERIMENTAL RESEARCH SAYING ABOUT MATHEMATICS WORD PROBLEM SOLVING? A REVIEW OF THIRTY-FOUR STUDIES COMPLETED BETWEEN 1980 AND 1986

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A problem is not necessarily 'solved' just because the correct response has been made. A problem is not truly solved unless the learner understands what he/she has done and knows why his/her actions were appropriate (Brownell, 1942).

How often do teachers accept a correct conditioned response for conceptualization?

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INTRODUCTION

In its Agenda for Action: Recommendations for School Mathematics of the 1980's, the National Council of Teachers of Mathematics strongly recommended that problem solving be the major focus of school mathematics. The NCTM recommendations were a result of studies conducted by the National Assessment of Educational Progress and the National Science Foundation. These studies revealed children's extremely poor performance in the problem solving skills.

The number of papers concerned with problem solving which have been published since the NCTM statement was made, abound. These efforts have been sincere responses to NCTM'S recommendation and deserve to be read and considered. The studies reviewed in this document, however, were limited to experimental research. They were published since 1980 when the NCTM recommendation was made.

The exclusion of experimental studies completed prior to 1980 in this document was because of resource limitations, especially time. Many of these prior studies were landmark contributions to the teaching of word problem solving. They too deserve to be read and considered.

CHARACTERISTICS OF VERBAL INTERACTION PATTERNS
OF TEACHERS OF MATHEMATICAL STORY PROBLEMS

Beth Ann Herrmann (1986)

Teachers who received identical training for teaching word problems to children and who taught these same strategies to children differed qualitively in their abilities to help their learners become strategic according to Beth Ann Herrmann (1986). The teachers differed in the ability to interpret their students' restructuring of information and spontaneously respond to student understanding or misunderstanding. They differed in their ability to stay on task when re-explaining. Those teachers who provided explicit explanations and interacted effectively with their students during instruction were more successful than teachers who provided explicit explanations but had difficulty interpreting their students restructuring of instructional information and spontaneity in responding to their students' understanding or misunderstanding. The teachers' ability to understand and explain the mental processing that occurred when using the strategy is more important than merely focusing on the procedural recall of the steps of the strategy.

Teacher educators should help their students see the relationship between teacher conceptual understanding and student conceptual understanding. Teachers need to know how to interpret

their students' restructuring of instructional information and spontaneously respond to student understanding or misunderstanding. Teacher educators should emphasize how to stay on task when re-explaining how to give students direct help in restructuring new instructional information. Teacher educators themselves need to learn how to interpret their own students' restructuring of information and respond effectively to students' understanding or misunderstanding. When students misunderstand, teacher educators must re-explain, re-model and give students direct help in restructuring the new information.

Comments, Criticism, Questions and Concerns

The researcher focused her study on the efforts of only two teachers. Did these teachers teach two separate groups? If so how did these groups compare in their general scholarly abilities? The researcher did not present her own explanations of the mental processes related to each step of the problem solving strategy process. An example of the step by step strategy process accompanied with example 'mental' processes for each step might have been helpful to the reader and to the teachers involved in the study.

STRATEGIC PROBLEM SOLVING OF MATHEMATICAL STORY PROBLEMS: A DESCRIPTIVE STUDY OF THE EFFECTS AND CHARACTERISTICS OF DIRECT TEACHER EXPLANATION

Beth Ann Herrmann (1986)

The efforts of four graduate students teaching twelve clinical students who were having difficulties with solving mathematical story problems were studied by Beth Ann Herrmann (1986). The purpose of her study was to determine the extent to which graduate students, who were trained to apply an explicit explanation model of instruction, provided such explanations when teaching poor problem solvers how to use problem-solving skills strategically and to describe characteristics of effective explanations. This purpose was based on research which suggested that students learn best when teachers are direct and explicit in providing instruction (Pearson, 1984; Roehler and Duffy, 1984; Brophy and Good, 1986).

The second purpose of the study was to examine the effects of explicit explanations about how to use problem-solving skills strategically on students' (a) awareness of how to be strategic when solving word problems; (b) conceptual understanding of the need to be strategic when solving story problems; and (c) the ability to select and apply specific strategies for solving story problems. The author based this purpose on descriptive studies of students' thinking which have examined the link

between cognitive processes and student achievement (Winne and Marx, 1982; Peterson, Swing, Braverman and Buss, 1982; Duffy, Roehler, Meloth, Polin, Rackliffe, Tracy and Vavrus, 1985).

The teachers received six hours of identical training on how to use techniques of explicit explanation when teaching mathematics story problem-solving strategies followed by an explicit explanation model.

Herrmann's results indicated that there was indeed a relationship between the explicitness of the teacher's instructional talk and what students learned, and that this relationship existed regardless of whether the instruction focused on strategic reading or strategic problem solving. The teachers who were more explicit in their explanations about how to use problem solving skills strategically were more successful in teaching poor problem-solvers to be strategic. The teachers who provided explicit explanations had students who were more aware of how to be strategic, better able to conceptually understand the need to be strategic and better able to apply problem solving strategies. Less explicit teachers had students who did not improve in awareness, conceptual understanding or ability.

Herrmann recommended that teacher educators put emphasis on how explicit explanations influence student awareness of how to be strategic, their conceptual understanding of the need to be strategic, and their ability to select and apply problem-solving strategies. Teachers need opportunities to plan and implement explicit explanations of how to use problem-solving skills strategically.

Comments, Criticism, Questions and Concerns

The population of the study involved only four graduate students (teachers') and twelve clinical pupils. How would the results differ if the population for each teacher was increased to 30 pupils? Did the author include an example of an explicit explanation model in the appendix?

COUNTING TYPES AND WORD PROBLEMS

Paul Cobb (1986)

Paul Cobb (1986) Purdue University conducted four analyses of models of childrens counting types. These included two computer-based models of arithmetical word problem solving ability and an analysis of developmental trends in children's solution methods to word problems. These analyses were Steffe, von Glaserfeld, Richards and Cobbs (1983) model of children's counting types and its extention to thinking strategies (Cobb, 1983), two computer based conceptual models of arithmetical word problem solving (Briars and Larkin, 1984; Riley, Greeno and Heller, 1983), and Carpenter and Moser's (1984) analysis of developmental trends in children's solution methods to arithmetical word problems. Cobb's analysis falls into two distinct parts. The first part relates to the work of Steffe

et al. and the two computer based models dealing with children's ability to solve various types of arithmetical word problems. The work of Carpenter and Moser was then focused on the nature of children's solution methods. The second part of the analysis sought possible relationships between Steffe and Carpenter and Moser's work.

Cobb's findings indicated relationships between the two computer based models and the work of the Steffe team with qualifications. Cobb also found a relationship between the Steffe model and Carpenter and Moser's analysis word problem solution methods. According to Cobb, these relationships did not determine which method a child utilized to solve a given task. They merely indicated the most advanced methods of solution that children typically used at given levels, eg., it was found that abstract counters generally used direct modeling methods for all change word problems. Cobb's findings supported the complexity of the problem with the open question, i.e., of how to sequence instruction in order to build on children's informal mathematical knowledge. Cobb's findings suggested that it might be premature to develop curriculum materials and instructional strategies explicitly designed to teach the use of specific methods of developing solutions. Cobb suggested a more cautious approach of developing activities and organizing classrooms to encourage children to develop self-generated solution methods.

Cobb's findings seemed to indicate differences in the ability of children at the same conceptual level to construct

and maintain representations of sensory motor actions.

Comments, Criticism, Questions and Concerns

Cobb's findings seemed to suggest a soft or cautious approach to some of the explicit instructional directions methods. The reader might conversely relate Cobb's conclusions to those of Herrmann (1986) concerning explicit instructions.

A respect for the work of Piaget might be suggested in this study.

COGNITIVE FUNCTIONING AND PERFORMANCE ON ADDITION AND SUBTRACTION WORD PROBLEMS

Thomas A. Romberg, Kevin F. Collis (1983)

Do children in the third grade who differ in cognitive-processing capacity solve addition and subtraction word problems differently? This is the question addressed by (Romberg and Collins, 1983).

The cognitive-processing capacity of children was related to the strategies they used to solve word problems (cognitive-processing theories are founded on the concept that mental functions can be described by the way information is stored, accessed, and operated on).

Eleven children who had just started the third grade were selected from 139 children who had taken 14 tests of cognitive functioning. Four tests were designed to measure working-memory capacity (M-space) for mathematical material. Ten quantitative processing tests were chosen.

Each child was individually interviewed on two sets of word problems by one of two trained interviewers on three occasions. An interview was defined as two sets of six problems each. The students were asked to solve these problems.

Romberg and Collis found that children differed in their cognitive capacity to function with quantitative verbal problems and that the children who differed in cognitive-processing capacity also differed in the strategies they used to solve given verbal problems and in their success in finding correct solutions. Further, children's decisions to use taught algorithms to solve these problems appeared to depend more on the semantic structure of the problems than on either instruction or cognitive capacity. These findings suggested that there are some children who are able to reason about quantitative problems, i.e., they know the basic procedures of addition and subtraction but may not use algorithmic procedures to find answers to verbal problems. They are satisfied to use direct modeling and counting. They are not convinced that the use of algorithms is efficient. Another group of children seemed to be weak in their ability to reason about quantitative problems. They did not know the basic procedures for addition and subtraction. They had not mastered the skills of direct modeling or counting

strategies. These children might try to use algorithms to solve verbal problems but not successfully.

Comments, Criticism, Questions and Concerns

Romberg and Collis suggested that we reexamine the relationship between instruction on algorithms and their application. They seemed to question the current emphasis on paper and pencil algorithmic procedures as being inappropriate at this early stage of development.

The population of this study seemed too small. The researchers outcomes seemed to support the work of Piaget. Their conclusions seemed sound and should be considered.

THE ROLE OF IMPLICIT MODELS IN SOLVING VERBAL PROBLEMS IN MULTIPLICATION AND DIVISION

Efraim Fischbein and Others (1985)

The basic assumption of this research was that arithmetical operations are intuitively associated with some primitive behavioral models whose existence and influence the person may not be aware of. These primitive models may act beyond any conscious control and may sometimes make the solving of problems easier or may slow down, divert or even block the solution

process when contradictions are formed between the model and the solution algorithm, i.e., that the models attached to arithmetical operations are basically behavioral in nature and that a person associates with a certain operation when trying to discover the intuitive model.

The Fischbein team (1985) hypothesized that the inactive prototype of an arithmetical operation may remain rigidly attached to the concept long after the concept has acquired a formal status. The team further hypothesized that the primitive model associated with multiplication is repeated addition and that the response for a problem situation which seemed to call for division would be partitive and quotative division.

The population included 628 pupils from 13 different schools in grades 5, 7, and 9. The pupils were attending school in Pisa, Italy. The students were given a 42 item test containing 12 multiplication problems, 14 division problems and 16 items were problems in addition or subtraction.

The results indicated that the initial didactical models of repeated addition for multiplication and partitive and quotative for division were so deeply rooted in the learner's mind that they continued to exert an unconscious control over problem solving behavior after the learner had acquired formal mathematical notions that were solid and correct.

According to the Fischbein team a fundamental didactical dilemma is faced by teachers. If they continue to teach the models of multiplication and division as described, strong resistant and incomplete models will be created which conflict

with the formal concepts of multiplication and division. If teachers avoid the behaviorally and intuitively meaningful models they will violate the most elementary principles of psychology and didactics. Teachers seem to know the problem but in line with Piagetian theory, they have assumed that learners would become less dependent on intuitive justifications and limitations and become more open to formal thinking in mathematics as they grew into their formal operations stage.

Comments, Criticism, Questions and Concerns

Could Fischbein have given examples of the desired formal mathematics behavior? In further research could he explore possibilities for encouraging the development of formal mathematics behavior? Could part of the problem be that we might often teach elementary mathematics without helping children develop an understanding of our number system? Do excessive exercises of drill and practice cause mathematical mind sets? Could some of the drill and practice be replaced by enrichment activities which enhance children's understanding of our numerical system, especially for some students?

DIAGNOSING STRENGTHS AND WEAKNESSES OF
SIXTH-GRADE STUDENTS IN SOLVING WORD PROBLEMS
Hunter Ballew, James W. Hunter (1983)

Ballew and Cunningham (1983) developed a diagnostic system for determining difficulties in solving word problems. Their system was used to classify 217 sixth grade students according to their strengths and weaknesses in solving word problems. The students were tested to measure their skills in computation, interpretation, reading and integrating these skills into finding a solution for a word problem.

The results indicated that each of the above four areas representated an immediate need for a sizable number of the sixth graders tested and that an inability to read problems was a major obstacle for sixth graders. The data indicated that weaknesses and strengths in the major areas of skills in solving word problems could be diagnosed through the use of a comparatively simple procedure.

Comments, Criticism, Questions and Concerns

The researchers admit a folly of trying to identify the problem that students in general have with solving word problems, i.e., any four of the areas of difficulty could be the problem depending on the student. The researchers suggested that further research was needed to develop and evaluate the effectiveness of treatment procedures for each of the four areas.

A suggestion to be directed to the authors might be to include the measurement of psychological as well as mathematical readiness skills.

THE EFFECT OF INSTRUCTION ON CHILDREN'S SOLUTIONS
OF ADDITION AND SUBTRACTION WORD PROBLEMS
T.P. Carpenter, J. Hiebert, J.M. Moser (1981)

The purpose of this study was to investigate the effect of initial instruction on the processes children use to solve addition and subtraction word problems. In an earlier study (Carpenter and Others, 1981) it was found that prior to formal instruction children were extremely successful when solving simpler addition and subtraction word problems.

The children in the study were individually interviewed in February of their first grade year prior to receiving any school instruction on addition and subtraction. The children completed ten verbal problems in addition and subtraction during the interview. After two months of instruction in addition and subtraction the children were given six of the same problems. More problems were given several days later.

The researchers found that prior to instruction the general strategy that most of the children used to solve addition and subtraction problems was to model directly the action or relationships described in the problem. The children had a number of different strategies for solving subtraction problems that represented distinct interpretations of subtraction. The researchers found that most children at this age did not recognize that the different strategies could be used interchangeably

to arrive at the same result. They seemed to regard them as distinct strategies that were used to solve different kinds of problems.

After the initial instruction, there was a distinct shift in the general approach children used to solve subtraction problems. Instead of using a variety of strategies to solve different problems, most of the children began to use a single strategy. It was not clear whether the children understood that several strategies were possible or whether their shift to a single strategy and a single unified interpretation of subtraction was caused by instruction. The results indicated that after several months of instruction children began to shift from a concrete direct modeling approach in solving word problems to a more unified conception which incorporated symbolic representation of addition and subtraction problems.

Comments, Criticism, Questions and Concerns

The number of children involved in the study was not given in the methods section of the study. The exact number of days later should have been given instead of merely "several". Is it fair for the authors to expect first graders to have a completely developed concept of subtraction? What about Piaget? How much do instruction and instructions contribute toward molding children into convergent thinking patterns and the use of single strategies for solving a variety of problems? Is convergent thinking a desirable outcome in the teaching of mathematics?

CONCEPTUAL UNDERSTANDING IN SOLVING TWO-STEP WORD PROBLEMS WITH A RATIO

Ana H. Quintero (1983)

The difficulties children have when solving multistep problems can be caused by difficulties in understanding the concepts and relationships or to difficulties in planning and organizing the method of solution. This study sought to probe the role of each of these factors in children's performance when solving two-step problems of a given structure.

Twelve children were selected from each of three grades 5, 6, and 7 in Puerto Rico. The researcher worked with one child at a time. Each session was one-half hour in duration and was audiotaped. Each child was asked to solve six word problems which were presented on cards one at a time. If the method the child was using to solve a problem was not clear to the researcher, the child was asked to explain the solution. After each problem was solved the researcher read the problem aloud and asked the child to repeat the problem from memory. According to the researcher repetition, if not done by rote, is a way to restate the problem as the child understands it. This kind of repetition can help the researcher identify children's representations of problems. After the problems were solved, the child was asked to choose from a set of drawings those representations of the situations described in the

two-step and the one-step problems. Some drawings correctly represented the problem situations others did not. A solution was considered correct if it was set up correctly even though there might be computational errors. A selection of one or more drawings was considered correct if it included only the correct representations. A repetition of a problem was considered correct if it contained all the information needed to solve the problem.

The majority of the children had difficulty with the two-step problems. Twenty-seven of the children had difficulty in understanding the concepts and relationships involved. Eighteen of these children did not understand the ratio given in the problem. The meaning of concepts and relationships in the two story word problem was a major source of difficulty in solving the problem.

Comments, Criticism, Questions and Concerns

The study seemed generally well structured. The researchers admitted, however, that the specific methodological difficulties the children were having were not clear. This may have been because of the difficult concepts involved. The authors suggested that such difficulties might be more effectively studied by having very simple concepts. Conceptual difficulties would then be minimized and the attention would focus on contrasting two-step and one-step problems.

A question directed to the reader might be: Why were these children having difficulties with simple ratios? Was

it a readiness problem or a developmental problem?

THE DEVELOPMENT OF SEMANTIC CATEGORIES
FOR ADDITION AND SUBTRACTION
P. Nesher, J.G. Greeno, M.S. Riley (1982)

The researchers suggested a hypothesis of developmental levels that could account for children's performances in solving given problems at a variety of ages. They postulated representational processes in children's understanding of problems corresponding to the derivations in their semantic analysis. This would explain the relative difficulty of different kinds of word problems. The researchers reviewed and presented empirical data for different categories of addition and subtraction word problems and proposed developmental levels of word problem solving ability that related to growth in empirical, mathematical and logical knowledge structures.

The results indicated consistent patterns of performance on 'change', 'combine' and 'compare' word problems involving addition and subtraction. The researchers tried to suggest a hypothesis that explained which kinds of problems could be solved without the use of arithmetic, and for which problems the knowledge of arithmetic was necessary. The researchers treated

the growth of the child's knowledge structure in a way that identified the empirical, logical and mathematical components. The researchers believed that their hypothesis and predictive power, could be examined empirically, and fit empirical data found to be universal.

Comments, Criticism, Questions and Concerns

The implications of the researchers' analysis indicated that teachers could be more sensitive to the sequence of instruction when they understood the prerequisite knowledge structures for solving certain problems. Different strategies could be adapted when teaching at different levels. The researchers' analysis increased our understanding of the difficulties that children encounter at each level of performance.

This study not only measured children's performance on 'change', 'combine' and 'compare' word problems but related psychological (developmental) factors to the performance. How often do teachers respect both performance and developmental factors when teaching mathematics?

The research was conducted in several countries. This paper did not seem to name the countries nor did it present the results according to each country. The total number of children involved was not given nor the number of children in each country.

THE EFFECT OF SYNTAX ON LOW ACHIEVING STUDENTS' ABILITIES TO SOLVE MATHEMATICAL WORD PROBLEMS

Larry J. Wheeler, Gaye McNutt (1983)

The purpose of this study was to determine the influence of syntax on the difficulty of word problems with 30 eighth grade students who were enrolled in remedial mathematics classes. These students were performing at two or more grade levels below their grade placement, but each had a minimal fourth grade achievement level in reading and computational mathematics. Three tests containing increasingly more difficult sentence structures were administered. These tests were written at a fourth grade level or below in reading vocabulary and mathematical computations.

The results indicated a difference between the test which contained the most difficult syntax and the two other tests even when the problems were at the students' computational and reading vocabulary levels. This finding implied that achievement tests in mathematics which included word problems could have been testing students' syntactic abilities as well as mathematical abilities. The researchers suggested that instead of rejecting books and materials with compound/complex syntax in favor of materials which are less difficult to read, that teachers should help students to develop the ability to break down complex-compound sentences. This would be practical and

beneficial to the students.

Comments, Criticism, Questions and Concerns

The purpose and method of this study seemed sound except for the small number of students involved and what seemed to be a lack of a group for comparison and a control group. Two questions arising after reading this study are: Should mathematics teachers also be teaching some language arts and reading? Should reading and language arts teachers also be teaching some mathematics?

EFFECTS OF COOPERATIVE, COMPETITIVE,
AND INDIVIDUALISTIC CONDITIONS ON
CHILDREN'S PROBLEM SOLVING PERFORMANCE
David W. Johnson, Linda Skon, Roger Johnson (1980)

The three purposes of this study were to: (1) compare the relative effects of cooperation, competition, and individualistic conditions on problem solving performance; (2) examine three possible influences on the problem-solving success of cooperative groups, i.e., quality of the strategy used, the benefits received by medium and low ability students interacting

with high ability group members and (3) the increased incentive to succeed resulting from peer support and encouragement for achievement. This study is another building block on the results of twelve previous studies (Johnson and Johnson, 1978).

Forty-five students in two first grade classes from a large mid-western suburb served as the population for this study. Two sets of independent variables were included in this study: (1) cooperative, competitive and individualistic learning situations and (2) three different learning tasks, i.e., a categorization and retrieval problem, a spatial-reasoning problem called the Rasmussen Triangle (Napier and Gershenfield, 1973) and a verbal problem solving task consisting of 10 story problems given or read to the students. The students participated in six instructional sessions of sixty minutes each given on different days, i.e.,

day 1, the students studied the story problems;
day 2, the students were tested on the story problems;
day 3, the students studied the Rasmussen diagram;
day 4, the students were tested on their abilities to identify the triangles in the diagram;
day 5, students arranged the words and tried to identify them;
day 6, the students were tested on their free recall, spontaneous retrieval, and awareness of the categories structure and search strategy.

The three teachers involved in this study were trained or experienced in demonstrating the procedures for teaching

cooperatively, competitively, and individualistically. The three teachers followed written directions for explaining the tasks.

Students in the cooperative condition outperformed students in the competitive condition on all four task measures. The most profound difference among conditions was on the categorization and retrieval task where four of the five cooperative groups discovered and used all four categories while no one in the competitive and individualistic conditions did so. Cooperative interaction seemed to promote perceptions of more support and encouragement for achievement than did competitive and individualistic conditions. The researchers concluded that when high problem solving performance based on the use of effective strategies and peer support and encouragement was desired, the instructional situation should be structured cooperatively rather then competitively or individualistically.

Comments, Criticism, Questions and Concerns

The results of this study have a direct practical application in classroom teaching. Life is a group effort and an individual effort as well. This includes the arts and sciences of teaching and learning.

The researchers' discussion on the results seemed to appear conflicting or confusing in the first and next to last paragraphs, i.e., the discussion on task measures and a comparison of the two conditions compared to the previous discussion.

There is a shortage of studies that include cooperative, competitive, and individualistic conditions in the same study. When is peer group cooperation not desirable?

QUESTION PLACEMENT IN MATHEMATICAL WORD PROBLEMS

Judith Threadgill-Sowder (1983)

The differing effects of pre- and post-questions in word problems was the major concern in this study. The hypothesis included the statement that prequestions should help students recognize other content and data as being incidental to the solution process.

The population included 52 students enrolled in two community college intermediate algebra classes. A set of 14 problems was designed for the criterion measure. The problems were varied to include 4 with extraneous data, 2 one-step problems and two step (or more) problems. The problems were written in two formats to accommodate the purpose of this study. In one format the questions were given at the beginning of the word problem. In the other format they were given at the end. Two analyses of covariance were performed. The Necessary Arithmetic Operations Test Score was used as the covariate measure.

No significant treatment effects were found for either extraneous data problem solving score or for non-extraneous data problem solving score. The results of the study indicated that question placement had no apparent effect on the ability of students to solve word problems. This was true regardless of the length and complexity of problems or the age of students.

Comments, Criticism, Questions and Concerns

The researcher's conclusion included students of all ages even though her study was done with college students. She mentioned similar studies completed by Williams and McCreight and Arthur and Clinton with elementary school children. Question placement effects were found to be non-significant in both studies and the problems which were used were found to be too brief to affect research behavior. Question to the researcher or future researchers: What are the effects of question placement on elementary school concrete operational learners? What are the specific elements of a word problem which can be tailored to increase its potential arousel and motivational potential?

COGNITIVE VARIABLES AND PERFORMANCE

ON MATHEMATICAL STORY PROBLEMS

Judith Threadgill-Sowder, Larry Sowder
John C. Moyer, Margaret B. Moyer (1985)

The researchers cite evidence that good and poor problem solvers approach such activities in different ways. The purpose of this study was to explore the relationships of certain cognitive variables to problem solving performance. The three cognitive variables involved in this study were spatial ability, cognitive restructuring and reading comprehension. These individual difference variables were examined in terms of how each is associated with different problem presentation modes.

The population sample consisted of children from eight classrooms in each of the grades three through seven located in eleven schools in differing socioeconomic areas of three mid-continent cities. A series of carefully selected cognitive tests were administered to the children in two sessions of up to one hour in each classroom. The tests were arranged in three formats giving each student three problem test scores, one for each format.

The findings indicated that story problems which were cognitively restructured so that drawings served to organize the data in the problem were most helpful to students scoring low on cognitive ability tests and yet were not detrimental to the students with high scores. The researchers hoped that this information might prove useful to teachers and textbook writers as well as to researchers.

Comments, Criticism, Questions and Concerns

The cognitive restructuring of text into drawings might serve as a bridge for learners who are in transition from concrete

operational thought to formal operational thought. This transitional tool might be most valuable for slow, average, or unimpeded learners. Four questions directed to research and teaching colleagues might be: (1) Are we moving into an era when conditions and tools for communicating demand that formal thought itself be redefined or cognitively restructured? (2) Should formal thought be redefined to include the cognitive restructuring of abstract concepts for slow and average learners, i.e., in practice, do our teaching methods and materials assume that learners become formal thinkers simultaneously? (4) Should formal thought be redefined or cognitively restructured for unimpeded and gifted learners to include advanced formal abstractions? An example might be a computer language such as logo which is considered to be an advanced form of abstract thinking for some children. Indeed, logo programming is thought by some researchers (Papert, 1980) to be capable of altering the thinking process itself when problem solving is involved.

The variables in this study were related to several previous studies completed by other researchers. This study could be recommended as an excellent example of research in education. The only obvious disappointment was the ommission of the total number of children involved in the study in the sample and procedure section of the method.

EFFECTS OF TEACHING SIXTH GRADE STUDENTS TO
MODIFY FORMAT VARIABLES OF MATHEMATICS WORD PROBLEMS

S. Alan Cohen, Georgia Stover (1981)

The researchers presented a trilogy of experiments which were concerned with the structural format variables of mathematics word problems which appear to hinder sixth grade students' abilities to read and solve these problems. Two of the experiments attempted to identify three format variables which interfere with student ability to understand math word problems. The third experiment instructed average sixth-grade students to modify these format variables as a way to increase understanding and the success rate for solving word problems. Previous studies were descriptive in that they implied the need to control the presence or absence of interfering variables. Previous experiments did not attempt to teach students the strategies necessary for overcoming the format variables which interfered with comprehension.

Children with a variety of abilities, grade levels and locations served as the population sample for these three experiments.

It was found that the effects of instruction were greater than the effects of aptitude. The researchers found it easy to teach sixth grade students to insert diagrams, to reorder

number sequences appropriately, and to remove extraneous information, thus improving their success rate in solving word problems. How long lasting are these results? The answer to this question could depend on the willingness of reading and mathematics teachers to adjust their curricula to include the three operations for modifying word problems and making them easier to understand. These strategies need to be included across the years.

Comments, Criticism, Questions, and Concerns

The researchers admit that these experiments incorporate a behavioristic definition of "understanding" operationally defined as "producing the correct solution to the word problem." To the behavioralist, controlling the comprehension of mathematics by manipulating antecedent conditions is both the essence of instruction and the "sufficient stuff" from which is developed a science of human behavior, according to the researchers. The reader's questions to the researchers might include: (1) Shall the learner's apply the operations out of shear habit? What happens when the habits are broken or the learners forget or are conditioned out of them? Are the researchers accepting a correct conditioned response for an internalized concept (conceptualization)? For how many days, weeks, years will the student retain the "behavioralistically" learned operations? Should teachers and publishers remove and edit-out any structural format faults which interfere with student success in solving word problems?

STRUCTURAL DIFFERENCES BETWEEN TWO-STEP WORD PROBLEMS

Valerie L. Shalin, Nancy V. Bee (1985)

This experiment tested the hypothesis that problem structure, or the overall pattern in which quantities are organized, is a significant factor in determining success in problem solving and the problem solving process itself, i.e., equivalence classes of problems may be defined on the basis of problem structure.

The population for this study included 82 third, fourth and fifth grade students. The grade assignment was determined by the textbook which each subject was using at the time of testing. The textbook levels ranged from grades three to seven.

The 82 subjects were presented with 34 word problems which were generated from 11 problem structures obtained by crossing triads of the operator type with problem structure. Most of the word problems could have been solved by using one of two competing conceptualizations. Word order was used to favor a simpler or preferred structure. Number set was crossed with problem structure to unconfound structural pattern with number facts. The subjects were grouped by textbook level and were tested in the classroom during a 40 minute session. The subjects were asked to do as many problems as possible, show their

work and to skip those problems which they were not able to do. The problems were scored likewise.

According to the researchers,the results of the experiment demonstrated the psychological validity of the structural analysis for two-step word problems, i.e., for all grade levels the problem structure, the overall pattern of relations between the quantities in the problem, had an effect on problem solving success. In essence, evidence was provided that different problem structures seem to define equivalent classes of problems and affect problem solving difficulty.

Comments, Criticism, Questions and Concerns

This study contained a substantial number of subjects compared to other studies. The researchers admitted that their results were not sufficient for clarifying the role of problem structure in the problem solving process but that there was good reason to support the belief that the problem structure effect is related to knowledge. According to the researchers, further analysis might lend support to the idea that each problem structure is an individual schema that guides operator selection and success in solving problems. The following are questions for the researchers and readers. If effective structuring of word problems to increase student success can indeed be achieved, should methods of effective structuring be standardized and taught to teachers and publishers? Are quantities in word problems always presented in numerical form?

FIELD-DEPENDENCE-INDEPENDENCE AND MATHEMATICS PROBLEM-SOLVING*

Sheila Vaidya and Others (1981)

The researchers, with a sample of 28 fourth grade subjects (14 boys and 14 girls), explored the implications of asking adjunct questions to the subjects at the end of a word problem.

The results indicated that the field-independent subjects solved more word problems correctly than did the field-dependent subjects. When the word problems were presented with adjunct questions there was some benefit to the field-independent subjects but a greater gain among the field-dependent subjects.

Comments, Criticism, Questions and Concerns

The researchers' work is related to many respectable previous studies completed by other researchers. The researchers suggested that the explorations of the strategies employed by field-dependent and field-independent children in solving mathematics word problems might prove valuable. Questions: (1) Is there a correlation between left or right brain hemisphere dominance and field-dependence or independence? Some studies indicate that successful mathematics students might be left-

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*A field-dependent person is known to be more competent than a field-independent person in situations that call for a social orientation.

A field-independent person is more successful in situations which require a high analytical ability, i.e., mathematics.

brain dominant. If so, are left brain dominant students field-dependent or field-independent? Should the development of a field-independent cognitive learning style be encouraged in learners? If so, how?

CHILDREN'S DIFFICULTIES WITH TWO-STEP WORD PROBLEMS

Ana Helva Quintero (1984)

This study included two experiments in which children's difficulties with two-step mathematical word problems were analyzed. Seventy-one fifth grade children were individually observed solving two-step word problems and one-step problems with the same mathematical structures as the components of the two-step problems in experiment I. The session required each child to solve five word problems in 10 to 20 minutes.

Most of the children (48 out of 71) had difficulty with at least one of the two-step problems. The children were more successful with the one-step problems. The children's difficulties seemed to be conceptual as well as strategic. Children tended to use the same strategies to solve two-step problems as they used to solve one-step problems.

Experiment II probed the hypothesis that if children were told beforehand that the problems given to them are solved using

two operations their performance would improve.

The population for experiment II had already participated in experiment I and was among the twenty-two children who had worked at least one two-step word problem incorrectly but worked its component correctly. The children's ages were between 9 and 11 years.

After the children completed the problems of experiment I, they were asked to solve two additional two-step problems with the same mathematical structure as the two-step problems that they had just solved. Half of the children were told that the extra problems were solved using two operations. The remainder of the children solved the extra problems as part of the original task.

A total of nine children (almost 41%) improved their performance on the two extra problems, six were in group 1 and three were in group 2. The children who worked a two-step problem correctly in the original task worked the corresponding extra problem correctly. Six out of eleven students (nearly 55%) improved their performance in the two-step problems after having been told that the problems required a two-step solution. Three out of eleven children (27%) improved their performance by just working the problems again. The results of experiment II confirmed the hypothesis that children's performance is affected by previous school experiences. A common error committed by children was to attempt to solve a two-step problem with a single strategy.

Comments, Criticism, Questions and Concerns

This study preceded and seemed to provide a foundation for the work of Beth Ann Herrmann. Again, as indicated in other studies, too many of the children approached two-step word problems with the strategies appropriate for one-step word problems. Indeed, whenever numerical quantities were given in two-step problems, the responses of a number of children was to add these quantities rather than to even consider subtraction or multiplication. Is this an example of children incorrectly applying a strategy which was once their correct conditioned response for solving other problems, i.e., are we as teachers correct in seeking correct conditioned responses as strategies for solving word problems? Were the children in these studies taught strategies for solving two-step problems before the tasks were administered? The population for experiment II seemed rather small. Quintero's efforts are respectable and have been complimented by other researchers.

INVENTED PROCESSES IN SOLUTION TO ARITHMETICAL PROBLEMS

James M. Moser, Thomas P. Carpenter (1982)

This study was concerned with the problem solving behaviors of primary age children on one-step word problems requiring

the use of addition and subtraction. The use of invention is defined as when children figure things out for themselves instead of applying formally taught facts and skills.

This study examined the inventive problem solving behavior of 100 first grade children during a three year longitudinal study. The three categories of inventive problem solving behavior included solutions which involved (1) place value; (2) algorithmic behavior and (3) the use of distraction of the child's attention away from the required algorithm, i.e., the child was caused to resist the use of the appropriate algorithm for solution.

The children were given four interviews during the three year period and were asked to complete six carefully selected word problems during each interview. The problems were worded in such a way as to prevent the subjects from immediately recognizing the problem type. Three units of mathematics instruction were taught to subjects in-between the interviews.

The researchers found that children used a variety of counting strategies to solve one-digit problems. The correct use of algorithms increased with instruction. A knowledge of place value seemed to appear to be necessary for the invention of algorithms in the sense that invention is a rearrangement of elements into similar structures. Instruction had a bearing on any changes in this invented behavior. Indeed, instruction had a major effect on the range and application of learned strategies. The findings also suggested that any characterization of children's formal or invented mathematics concepts

and procedures should consider the role of instruction.

Comments, Criticism, Questions and Concerns

The researchers' discussion of the inventive problem solving behavior involving the use of place value seemed limited in the findings and discussion of the study.

How much did the use of distracting semantic structures, designed to "camouflage" the needed subtraction algorithms as addition, contribute toward instilling misconceptions, frustration, confusion and math anxiety in these very young children?

Why doesn't more of the research concerning wording focus on older children in grades five through nine?

How might instruction increase children's effectiveness in solving problems without inhibiting the development of their divergent (inventive) mental processes?

How important is it that first graders do word problems? second graders?

PROBLEM SOLVING STYLES AMONG CHILDREN WORKING IN SMALL GROUPS ON MATHEMATICAL WORD PROBLEMS

Kathleen Gilbert, Steven Leitz (1982)

The most prevalent difficulty encountered when studying

problem solving processes of young children is that we can see the input and output, but we cannot see the actual processing, i.e., the mental operations that the children use to achieve a solution. Children working in small groups express a variety of problem solving styles and strategies which are sometimes concealed during ordinary classroom activity. The purpose of this study was to focus on four children, each of them working in a different group of four eleven and twelve year old fifth and sixth graders. Their problem solving discussions were recorded during four twenty-five minute meetings. These groups met weekly away from their regular classes. The purpose of the researchers was to generate hypotheses rather than to verify theory.

Among the generated hypotheses was the statement that work in small groups helps children to be more effective in mathematics problem solving.

Among the findings were two aspects: (1)In over 90% of the problems completed by small groups, a solution was available within three acts of the reading of the problem; and (2) In all of the groups examined there was always one student who was credited with less than 10% of the total acts. In every group one student gave over half the answers to the problems while another member gave fewer than two answers.

Comments, Criticism, Questions and Concerns

If indeed, participation in small groups helps children become more effective problem solvers, is it because of the

small group dynamics at work? the peer group instruction? or a combination of both?

Do only students who are active participants in the group experience gains? Can gains in problem solving skills be made by "silent" participation in the group? How much does the emerging group 'pecking order' determine one's success in the group? What are the factors which determine the 'pecking order' within the group, i.e., status and sociability, math and verbal ability, sex?

DRAWN VERSUS VERBAL FORMATS FOR
MATHEMATICAL STORY PROBLEMS
Judith Threadgill-Sowder, Larry Sowder (1982)

The major question for this study was: "Might problems presented through drawings result in better student performance than would problems presented only in verbal form?"

The population consisted of 262 students in ten fifth grade classes in Calgary, Alberta, Canada. Four sets of 32 problems in verbal and drawn formats were developed. Five classes were given the verbal treatment and five the drawn format. A 16 problem post-test was given.

The data supported the hypothesis that students are

generally helped by drawings used as a vehicle for presenting problems. Presenting "word problems" by way of drawings was clearly more effective than the standard words-only presentation for the students tested.

Comments, Criticism, Questions and Concerns

The researchers regarded the results of this study important from instructional and research viewpoints. The drawn format was preferred by students and served as a reasonably close simulation of many real life problems; especially those in the marketplace.

The data did not support some of the researchers' other questions, i.e., How are field independence, spatial visualization and general reasoning ability related to student performance on solving problems in the verbal and drawn formats? Are there any aptitude-treatment interactions? The answers to these questions seem worthy of further pursuit.

The study of the effectiveness of drawings to communicate the content of mathematics word problems and increase performance in finding solutions is valuable. Is it possible to substitute still photographs for drawings? What about three dimensional 'stills' with children taking turns modeling and developing the 'sets'? What about dramatizing each word problem with children writing the scripts, making the costumes and 'sets'? This would combine mathematics with language arts, reading, design, measurement and crafts.

THE USE OF SMALL GROUP PROTOCOLS IN ANALYSIS OF CHILDREN'S ARITHMETICAL PROBLEM SOLVING

Nel Noddings (1982)

This was a clinical study. The major purpose was to observe the discourse of 5th and 6th graders solving word problems in small groups without teacher intervention.

The study included 24 students (boys and girls) performing at below grade level in mathematics and reading. Each of the six groups of children completed eight word problems in each of four sessions. The problems were carefully selected so that their computational and reading requirements were somewhat behind the children's measured achievement scores. The children were asked to talk outloud during their group task involvement, i.e., reflexive talk was encouraged.

The clinical process revealed specific conceptual difficulties which the children were having. According to the researcher, the small group working without teacher intervention seemed to be a powerful learning tool. Each group completed its fourth and final session with perfect scores. The researcher observed genuine child engagement and pleasure in the small groups. The obvious weakness observed and reported by other researchers was the case of the socially withdrawn child who spoke only when addressed.

Comments, Criticism, Questions and Concerns

Most of the comments concerning the work of Kathleen Gilbert and Steve Leitz (1982) apply to this study, eg., the power of peer group instruction. It was interesting to note the problem that stymied the children:

> James ate one-third of a pizza. How much did he have left?

The study suggested that difficulties with fractions are widespread. The concepts related to fractions seem to especially necessitate the applications of Piagetian principles by the teacher. The language of fractions is especially abstract to children who are mostly limited to their confines of concrete operational thinking processes.

Do small group methods result in improving individual problem solving achievement? The researchers hoped to address this question at the next stage of their research.

A COMPARISON OF CONCURRENT AND SEQUENTIAL INSTRUCTION OF FOUR TYPES OF VERBAL MATH PROBLEMS

Eric D. Jones, James P. Krouse, Donna Feorene, Carol A. Saferstein (1985)

The purpose of this study was to compare the relative

effectiveness of two structural variations of a direct instruction procedure. This direct instruction procedure was for teaching children a strategy for recognizing the difference between addition and subtraction story problems.

The population of 29 children was selected from 142 pretested third grade students on the basis of having failed to select the correct operation for at least 25% of the items. The selected children were then randomly assigned to either sequential training or concurrent training.

The same training problems were used in both the sequential and concurrent conditions. The students were actively engaged in instruction for nine periods of fifteen minutes each.

The results of this study indicated that the third grade children trained to solve four basic types of verbal math problems in a sequence achieved higher post-test scores than those students who were trained to solve an unsequenced arrangement of the same problems. The sequentially trained students initially had significantly lower pretest scores and significantly higher post-test scores than did students in the concurrent training conditions.

Comments, Criticism, Questions and Concerns

The sequencing of any program lends itself to repeated responses to similar relevant attributes and would increase the probability that students recognize these basic attributes and respond appropriately. Is this kind of programming really

a conditioning process? If so, for what periods of time shall the students retain what is learned? Which specific teaching methods were used in each group? Were the meaningful applications of computational operations to manipulations of concrete objects and events used (the researchers cite Cawley's work of the 1960's and 1970's)? If not, how much more effective might have these efforts been toward the positive long term effects, i.e., conceptualization and retention, rather than the limiting board and seat work materials used in this study for both conditions? The design of this study seemed to lend itself more toward the testing of two types of curriculum materials, rather than teaching strategies, i.e., the two training conditions seemed to be defined in terms of curriculum materials rather than different teachers' strategies. Why not a study comparing the rote, repetition conditioning methods with the demonstration of concepts through the use of concrete objects and events?

COGNITIVE DEMANDS THAT ARITHMETIC
WORD PROBLEMS IMPOSE ON CHILDREN
Denise K. Muth (1982)

The researchers in this study hypothesized that: (1) computational ability and reading ability each account for

significant amounts of variance in the accuracy of solutions and (2) the presence of extraneous information could strain the learners' limited processing capacities and reduce the accuracy of their problem solutions.

The population for this study was defined as 200 sixth grade girls (109) and boys (91) from two schools located in a university community. The two individual difference variables were students' reading ability and their computational skills. There were two format variables: (1) extraneous information and (2) syntactic structure. The Comprehensive Tests of Basic Skills (1976) and a 15-item word problem test were the experimental materials for this study. Four test versions were randomly assigned to the 200 students in eight sixth grade classrooms. An equal number of students received each version.

The data supported the hypothesis that reading ability played a major role in the solution of word problems and that extraneous information reduced the accuracy of students' answers and increased the length of their test-taking times.

Comments, Criticism, Questions and Concerns

This study can be considered an effective example of educational research. The researchers questioned the findings and conclusions of previous researchers and produced data which supported opposing views. The statistical treatment of the data seemed competent. However, is it possible that the subjects participating in this study were skewed somewhat in their skills and performance? They were located in a university

community which indicated the possibility of a large number of middle class learners.

The researchers encouraged teachers and textbook writers to design activities which help learners to integrate their basic reading and mathematics skills and to effectively apply them.

Should the teaching of reading be combined with computational skills as a method for developing and assessing comprehension while simultaneously teaching and reinforcing problem solving skills?

INSTRUCTIONAL APPROACHES AND LEVELS OF PRACTICE IN WORKING FOURTH GRADE WORD PROBLEMS

Craig Darch and Others (1983)

The purpose of this study was to intensively look at the aspects of mathematics instruction, development and practice, i.e., to contrast the direct instruction approach to the development phase of teaching with a traditional approach. The traditional approach was taken from the teachers' manuals of four widely used state-adopted textbooks. The solution of word problems was selected as the major topic because it was a common target area for students in the intermediate grades and an area

for which many philosophies have been developed. Specifically, the topic was the teaching of multiplication and division story problems to skill deficient fourth graders. The second purpose (and variable) for this study was the provision of extended practice for low performing students.

Seventy-three students were chosen as subjects because they failed the mathematics story problems screening test given to the 220 fourth graders in six regular classrooms. Each classroom teacher then verified that these students were having difficulties with word problems. These students then had to demonstrate computational abilities in addition, subtraction, multiplication and division in order to participate in the study. The subjects were almost exclusively from middle income families and were taught with one of the four state adopted basal mathematics programs.

The 73 students were assigned to one of four experimental groups: (1) direct instruction with a fixed amount of practice; (2) traditional instruction with fixed amounts of practice; (3) direct instruction with extended practice; (4) traditional instruction with extended practice. The students were usually taught in small groups in their regular classroom.
The experimental teachers received training for each of the experimental conditions. Three tests of problem solving and one consumer questionnaire were developed for this study. The problem solving tests included a 40-item screening test, a 26-item post-test and a 26-item maintenance test. Five lessons were audiotaped for each experimental teacher.

The direct instruction approach resulted in higher posttest scores. The data supported the position that a program designed to teach (1) prerequisite skills in a sequential manner and (2) explicitly teach problem solving skills was superior to a traditional approach with these subjects. It was apparent that a mere increase of instructional time without altering instructional procedures and curriculum was not sufficient to produce changes in the performance of these lower performing students, i.e., time on task alone did not lead to increased performance. How the students used time and what was thought while studying was important.

Comments, Criticism, Questions and Concerns

The findings in this study were similar to those in other studies which supported the use of direct sequenced instruction for teaching the prerequisite skills and problem solving strategies. This resulted in higher performance in solving word problems. Traditional textbooks, curriculums and teaching strategies seem to omit these kinds of prerequisites for teaching word problem solving, why?

The subjects in this study were almost all from middle income families and all of them could already add, subtract, multiply and divide. Might the results differ in more mixed situations or even in inter-urban or extra-rural situations?

Did not the very design of this study allow students additional time in the word problem solving arena and thus affect their abilities for solving them? Does the very act

of investigation change the character or process of that which is being investigated (Harris, 1986)?

What are the specific prerequisites for success in solving word problems? Why are these specific prerequisites omitted from so many of the traditional textbooks, curriculums and teaching methods?

A possible paradox: Does the entire education effort unintentionally 'stack the deck' against word problem solving then laments the results and pleads for solutions, a victim of its own size and complexity which cannot always readily change itself?

THE EFFECTS OF ADJUSTING READABILITY ON THE DIFFICULTY OF MATHEMATICS STORY PROBLEMS

Douglas J. Paul, William H. Nibbelink, Hiram D. Hoover (1986)

In the opinions of the researchers, previous investigators did not accurately isolate readability as the independent variable and therefore the question of the effect of readability levels on story problem difficulty was left unanswered. The purpose of this study was to more carefully isolate readability as the independent variable by holding other factors constant.

The major question was whether the use of popular readability formulas provided valid information toward deciding if word problems were appropriate for a given grade level.

A total of 1,238 students from seven Iowa schools in grades 3, 4, 5, and 6 were selected for participation in this study. Six test forms were randomly assigned to students in each grade. The fifteen computational problems were especially designed to be representative of the computations required for solving word problems in widely used textbooks and standardized tests. To determine the readability of an item the Harris-Jacobson Formula 2 was applied. Formula 1 was applied for even greater accuracy.

Data were analyzed by a mixed fixed-effects four-way analysis of variance. The factors were readability level, problem type, grade and readability adjustment method.

The findings of this study indicated that whether a word problem had a 'readability score' a few grades above or below grade level there was no substantive effect on students' ability to solve it. Thus, the practice of applying readability formulas as a factor in determining the grade level appropriateness of story problems was not supported by the results of this study. Indeed, it conveyed an admonition to avoid the application of formulas to uses for which they were neither designed nor intended.

Comments, Criticisms, Questions and Concerns

This study seemed like an example of outstanding educational

research. The controls and statistical tools were carefully chosen and designed into the study. The size of the sample (1,238) lends credibility to the findings.

The negative comments about particular applications of readability formulas made by the researchers in the very beginning of the study may be interpreted to lend support to their own eventual findings. The comments might also be interpreted as more than a hint of bias or pre-judgment against particular applications of readability formulas. Perhaps the contents of the paragraph might have been more appropriately placed in the findings or conclusions. These results, conclusions or opinions supported those of Fitzgerald and Cullinan (1984).

THE REPRESENTATION OF BASIC ADDITION AND SUBTRACTION WORD PROBLEMS

Thomas P. Carpenter and Others (1985)

The purpose of this study was to investigate children's representation of a wide range of problems involving addition

and subtraction with canonical and noncanonical number sentences.* The primary concern of this study was whether or not children would directly represent the structure of a problem if both canonical and noncanonical number sentences were available for such a representation. The major objective was to identify the number sentences that children in each group used in order to solve a variety of addition and subtraction problems. The researchers' purpose included the examination of the relationship between these number sentences which the children wrote and the semantic structure of the problems. It seems that the semantic structure of word problems plays an important role in children's strategies for solving problems at this age using modeling and counting techniques. Thus, it was hypothesized that the structure would significantly influence the number sentence representations of the children in both groups.

The population for this study was made up of 22 first-grade children and 41 second-grade children from a predominantly middle class area of Madison, Wisconsin. The subjects had only a limited exposure to solving word problems prior to this study. This exposure consisted of simple joining and separating situations. No prior instruction had been given on writing noncanonical number sentences or on writing complete open sentences with

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*Note: examples:
canonical ($a + b = \square$, $a - b = \square$)
noncanonical, ($a + \square = b$, $a - \square = b$,
$\square + a = b$, $\square - a = b$)

This study was not designed to investigate the relative effectiveness of different instructional treatments.

a box to represent the unknown.

The children were randomly assigned to either a canonical group or a noncanonical group. Each group received two 30-minute periods of instruction on writing and solving number sentences and on writing number sentences to represent word problems. Following instruction the children were tested on their ability to write number sentences to represent addition and subtraction word problems. In the first part of the test the children were instructed to write a number sentence for the problem and to solve the problem. In the second part of the test the children were instructed to write only an appropriate number sentence and not to solve the problem. In this part of the test the children's responses were so guided by the design of the test to cause them to write open number sentences with a box to represent the unknown. The test was administered to children, from canonical and noncanonical groups combined, in their regular classrooms.

In the results of the first graders, in spite of the fact that children in the noncanonical group had only been instructed on representing joining missing addend problems, over 65% of them attempted to directly represent the action taking place in these four problems. No inappropriate noncanonical sentences were written and for certain problems no children in the noncanonical group wrote a canonical number sentence. Children on this level most naturally represented problems directly and although they had received no instruction in the direct representation of problems, as many as 45% of them attempted

to write number sentences that represented the action described in these four problems.

The second graders were generally more successful than the first graders in representing and solving the more complex problems and in transforming problems to represent them as canonical sentences, eg., over 75% of the second graders in the canonical group wrote correct canonical number sentences for five of the six join and separate problems.

The results of the study suggested that most of the first graders were limited to direct symbolic representations of word problems but that first graders could learn to write noncanonical number sentences and use them to directly represent the action in appropriate problems.

The implications of this research for instruction include: (1) the limiting of exposure to those types of problems which correspond to canonical number sentences until children attain a level at which they can transform problems (at the risk of developing a rather narrow perspective on the types of problems)*; (2) instruction which includes noncanonical as well as canonical number sentences more nearly corresponds to the learner's natural representations and solutions of word problems than to immediately represent all problem situations in canonical form; (3) instructions involving noncanonical number sentences should not be dismissed because of any presumed

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*Note: This condition is so temporary in the child's development that this risk may be worth taking.

past failures; (4) instruction which includes noncanonical number sentences seems to be a viable approach for buiiding on the informal number concepts and skills which children possess prior to instruction.

Comments, Criticism, Questions and Concerns

This study was carefully thought about, planned, designed, and carried through to completion. The results seem to support some of the contributions of developmental psychology, i.e., children pass through several levels in the development of addition and subtraction concepts and skills. These levels apply to symbolic as well as physical representations of problems, i.e., children in the initial levels are able to only represent problems symbolically with number sentences when the sentence corresponds to the action in the problem.

Once again as, in other studies, the use of three dimensional models to represent the problem to concrete operational thinkers is omitted as a possible variable.

The total population sample was 63. This might be considered too small for some readers of the study. The subjects were middle class and lived in a city containing a very large university. How might the results differ in an inter-city or a rural setting?

The results were reported in readily consumable form with the use of simple percentages. Thank you.

According to previous researchers, i.e., Copeland (1979), first graders do not possess the psychological tools for solving

such mathematical sentences as 3 + ☐ = 5. These algebraic sentences are presented prematurely to young children.

ESTIMATING THE OUTCOME OF A TASK AS A HEURISTIC STRATEGY IN ARITHMETIC PROBLEM SOLVING: A TEACHING EXPERIMENT WITH SIXTH-GRADERS (1981)

Erik De Corte, Raf Somers (1981)

In this study the researchers hypothesized that the difficulties of sixth graders with word problems were mostly caused by a lack of the thinking procedures for analyzing and transforming a problem to a familiar form representing a routine task. To improve sixth graders problem-solving ability, therefore, learners should acquire the attitude and the skills to analyze and represent the data in the problem. A second hypothesis stated that the systematic estimation of the outcome of a word problem before working out the solution would prove to be an effective heuristic strategy for encouraging the learner to analyze the problem and to anticipate its solution, i.e., that passing roughly and in an abbreviated form through the solution process as is required when estimating an answer, helps the learner achieve a solution. The estimating process encourages the learner toward some degree of problem analysis

causing the problem and its possible solution to become more transparent. Thus one of the major purposes of the study was to teach children a strategy for solving word problems in which their estimation of the outcome before attempting a solution became central.

Tests were administered to the experimental and control classes.

The findings suggested that these sixth graders rarely applied systematic problem analysis when confronted with a word problem. It was quite common for them to begin computations immediately upon reading a problem. The findings revealed an almost complete lack of a solution verification process by the learners. They did not even try to check their answer for accuracy or plausibility. Needless to say the findings supported the hypothesis that learners difficulties with word problems were caused by a lack of the attitude and skills necessary to analyzing a problem before attempting to compute a solution. The hypothesis that teaching learners estimating as a solution procedure will increase their ability to solve word problems was supported by the findings.

In summary the findings seemed to support the belief that the sources of the numerous complaints about the poor results of instruction on word problems should be sought within those teaching methods now practiced.

Comments, Criticisms, Questions and Concerns

This study was completed in Belgium and the reseachers

might not have been American. In our own words, the heuristic estimation strategy encouraged the learners who participated in this study to go through a "dry-run" of the solution process before attempting a final solution to a word problem. Estimating answers to algorithms has long been a part of the mathematics curriculum and strategies of teaching. DeCorte's applications of estimation as a strategy to improve learner's word problem solving abilities might be creative if not unique.

It was made clear that the subjects in this study were sixth graders and that their number was small. The total number of subjects did not seem apparent in the report.

DeCorte contributed a hypothesis for further study as well as suggestions for conducting the research. These are included in the section containing hypotheses and questions for further research.

Was it the estimation of the outcome itself that made a difference in DeCorte's learners' word problem solving abilities or was it the abbreviated "passing through" the solution process, i.e., the "dry-run" which was encouraged by the estimation? What other factors could be applied to encourage the "dry-run" through a word problem?

THE EFFECTS OF LANGUAGE AND SCHOOLING ON THE SOLUTION OF SIMPLE WORD PROBLEMS BY NIGERIAN CHILDREN

Lawal O. Adetula (1985)

There were three major purposes for doing this study: (1) to verify with Nigerian children models of the knowledge and strategies which underlie children's solutions to basic addition and subtraction word problems (Briars and Larkin, 1984; Riley et al., 1983); (2) to measure the influence of language on children's ability to solve simple arithmetic word problems, and (3) to study the effect that schooling has on children's solutions of basic addition and subtraction word problems.

The population sample for this study consisted of 48 schooled and 47 unschooled children. The schooled children were randomly drawn from the Yoruba speaking groups in grades one through four of a Nigerian University Staff School in Zaria. Twelve students were drawn from each grade level. The 47 unschooled subjects were randomly drawn from the community where the staff school was located. Their ages were seven to fourteen years. These children could speak Yoruba but had little or no schooling.

Individual interviews were used to identify the process which the children used to solve each of the problems. During the interviews the schooled subjects were asked to solve 15 addition and subtraction word problems in English and 15 in

Yoruba. The unschooled participants were asked to solve 15 problems in Yoruba.

The findings indicated that simple join, combine addition, and simple separate were the easiest problems to solve for all the participants in this study. The compare problems and the separate start unknown problems were the most difficult. Thirty six of the 48 children in the schooled sample performed better when the problems were in Yoruba. Five students performed better when the problems were presented in English.

The results indicated that unschooled children were, in the final analysis, as successful in solving word problems as the schooled children, but that this success was true of the older unschooled participants only.

There were three major conclusions to this study: (1) Problem solving skills which were not very different from those in western cultures were developed in Nigerian children in addition and subtraction. The abstract counting strategies used by western children were less evident with the participants of this study. (2) Problem solving skills were not a total function of the schools; the performance of the unschooled children on addition and subtraction word problems was similar to that of the schooled children. (3) The language of problem presentation had an effect on performance. The children performed better when the problems were presented in their native language, Yoruba.

Comments, Criticism, Questions and Concerns

The unschooled children in this study and in previous similar studies in Africa were members of families who were traders and merchants. Their mathematical abilities were obtained from their daily activities in the marketplace. These activities included the handling of money based on the decimal system and buying and selling goods which were packed in dozens.

The exact number of interviews and the duration of each was not apparent in the procedures. Were the problems presented in written form or orally to the unschooled participants?

The term unschooled is used somewhat loosely in this study. The unschooled were "schooled" in the marketplace. This study was an interesting source of information as to where young persons in other cultures acquire their problem solving skills outside the realm of formal schooling. Perhaps similar studies are needed in other world locations where people survive and even thrive with little or no formal schooling.

CONCEPTUAL UNDERSTANDING IN SOLVING MULTIPLICATION WORD PROBLEMS

Ana Helvia Quintero (1981)

The main purpose of this project was to study the role of mathematical conceptualizations and other factors involved in

solving a word problem.

The study required two experiments. Thirty-six children of Hispanic background between the ages of eight and twelve years participated in the first experiment. The word problems were presented to the children in the language of their preference, Spanish or English. The children met in groups and individually with the researcher. They were audiotaped during game playing, task performance, and problem solving.

In the second experiment the participants included 24 children of Hispanic background. All of them could speak Spanish but some of them preferred to read their materials in English. Tasks were undertaken with only one child at a time over a 30 minute period. The youngsters were given a concrete representation of the problem to minimize the difficulties of interpreting it. Thus, it could then be tested whether or not the children could map this representation of the problem and relate it to a mathematical operation which shared a like structure. A posttest which was equivalent to the pretest was given at the end of the experiment.

The children were asked to restate each problem as they understood it. There was a high correlation between the problems solved and restated correctly. It was found that the children rarely repeated the problem in the same words as it was read to them even when they restated it correctly.

The results suggested that the major source of difficulty was in repeating the intensive quantity. This supported the hypothesis that the representation of the intensive quantity

was particularly important in solving these problems. The only quantity that was varied was the embeddness of the intensive quantity.

Contrary to what was expected there was a trend for the abstract problems to be the easiest for the children to solve.

In the second experiment the most common error was the failure of the children to represent the relationship between the elements that form the intensive quantity, i.e., the children had difficulty in exercising an ability to understand the relationship between the elements that enter to form an intensive quantity.

The researchers emphasized the importance of precise typology to characterize the word problems which children are asked to solve in order to better understand children's difficulties in solving multiplication word problems.

Comments, Criticism, Questions and Concerns

Research in Education is conducted in order to generate information that is consumable by teachers for the purpose of making a difference in the classroom. Many teachers are not statisticians. It would have been helpful to the reader-teacher if the definition of terms page had been designed to give a clearer, more meaningful explanation of these terms: intensive quantity, typology, embeddness. A simple table could have served as a 'key' to more clearly defined and expanded meanings of important terms and their symbols rather than dependence on text and outline alone. This would go hand-in-hand

with and be supportive of what might have been the researchers' most important statement: the improvement of children's performance when solving intensive X extensive (IXE) problems after working with drawings and concrete materials suggests that the use of these two and three dimensional manipulative devices or multiple embodiments to represent the most basic elements of word problems can be an effective instructional aid.

It is possible that the other most important finding stated by the researchers concerned the high correlation between problems worked and those restated by the child correctly, i.e., those word problems the child restated in his/her own words and the way he/she understood them. These findings combined with alleviating the children's difficulties with understanding and restating the intensive quantity in a word problem are vitally important contributions to children, teachers and the total efforts toward the improvement of problem solving. This reader recommends that efforts be made to communicate these three important findings to teachers and commends the researchers.

The children had difficulties with understanding the concept of ratio, i.e., they lacked a conceptual understanding of one of the components of the word problems presented to them. The statements of hypotheses were not readily apparent before the explanations of the methods and procedures.

The diagrams of the word problems and the table in the appendices were commendable.

The investigators suggested that this research can be expanded to include the study of how children work one-step

multiplication problems of different structures.

AN EMPIRICAL VALIDATION OF COMPUTER MODELS OF CHILDREN'S WORD PROBLEM SOLVING

Erik DeCorte, Lieven Verschaffel (1985)

The major purpose of this study was to use data derived from a longitudinal study to test the various predictions derived from the computer models of elementary arithmetic word problem solving developed by Riley et al (1983). The researchers focused on these aspects of the Riley team's analysis of the solution processes in the change problems: (1) the answer patterns of the individual children on the different types of change problems; (2) the appropriate problem representations; (3) the correct solution strategies; and (4) the nature and origin of the children's errors.

The subjects for this study were 30 first graders who were interviewed individually three times during the school year. Each time they were administered a series of Piagetian tasks, memory tasks, counting tasks and eight word problems.

The researchers' findings support a great number of predictions of the Riley team's computer models concerning the sequence in which the different problem types were mastered, eg., the 30 children correctly solved the change 2 problem in the beginning of the school year and none of the children who responded to change 3 problems correctly failed on change 1 or 2 in accordance with the Riley team's predictions.

The observations of this research team revealed a change in the children's solution strategies as the school year progressed, eg., verbal and material strategies in the beginning of the school year and a marked increase in mental strategies in the end of the year.

In conclusion the researchers also had important findings which were not in agreement with the Riley team's models. Among the reasons offered by the researchers to explain why the computer models did not account for many of their findings was the fact that the Riley models were strongly based on a rational analysis of the simulated cognitive processes and less on empirical data. Secondly, the descriptive and explanatory data from the Riley models decreased in value as soon as the formulation and the mode of presentation of the problems ceased to coincide completely with the corresponding aspects in the computer models. The third factor concerned the text-processing component. The description of the variables and processes which contributed to the construction of an appropriate representation of the problem text was not sufficiently elaborated in the Riley analysis. Thus, several appropriate and inappropriate problem representations

were completely lacking in the computer models.

The researchers did not deny the value of the computer simulation as a tool for research in word problem solving, eg., the computer is a very appropriate tool for modeling cognitive structures and to process underlying intellectual performances. A computer represents structures and processes explicitly and unambiguously, making it an excellent starting point to generate hypothesis which can and must be verified empirically.

Comments, Criticisms, Questions and Concerns

The DeCorte team applied self-report techniques and the observation of behavior to test the series of predictions from Riley's (1983) models, i.e., qualitative empirical data gathered by and from living subjects used to test the adequacy and validity of computer models. The value of data collected by human beings observing the behavior of other human beings might surpass that of computer models indefinitely. The reader should note, however, that at any given time computers are limited by the state of the technology and existing programs.

In the beginning of the study the researchers outlined an excellent history and timeline of the research in word problem solving including the researchers' names, their contributions and the year that their work was presented.

What specific contributions toward the improvement of word problem solving are coming from the realm of logo? Which kinds of problem solving behaviors or models might be effectively tested by computers?

THE ROLE OF SEMANTIC UNDERSTANDING IN SOLVING MULTIPLICATION AND DIVISION WORD PROBLEMS

Judah L. Schwartz/Ana Helvia Quintero (1981)

The researchers were concerned with the semantic aspects of word problems, i.e., the meaning of the concepts and their relationships involved in the problems. This study was intended to further the understanding of how children solve word problems by studying the difficulties that children demonstrate when solving two-step multiplication problems of a given structure.

Two experiments were included in the study. The first experiment involved 36 children between the ages of 9 and 14. Each child and the experimenter went through two tasks at a time. The children were asked to solve a set of word problems and to choose from a set of drawings the situations represented in the problem.

The subjects for the second experiment were 93 children between the ages of 9 and 13 years in grades 5, 6 and 7. Each child was given a set of four two-step problems to solve and asked to write the operations used to solve the problems.

The results of the first experiment indicated that a major source of difficulty the children had with the word problems was in inderstanding the relationship between the elements that form a ratio. A second source of difficulty was in understanding the

relationship between the ratio and the extensive quantity.

The second experiment revealed that the concept of ratio continued to be a source of difficulty for children as was the two-step word problem.

Comments, Criticism, Questions and Concerns

The clear and precise statements of hypotheses questions or aims for each of these experiments were not readily apparent.

What was the nature of the population sample in terms of abilities?

In which language were the problems presented to the children?

Do children have difficulties with the concepts of simple ratios and fractions for some of the same reasons they have difficulties with word problems?

USING THE MICROCOMPUTER TO TEACH PROBLEM-SOLVING SKILLS: PROGRAM DEVELOPMENT AND INITIAL PILOT STUDY

James M. Moser, Thomas P. Carpenter (1982)

The purpose of this report was to describe the results of the initial phase of the microcomputer research project conducted

by the Mathematics Work Group of the Wisconsin Center for Education Research. The aim of the project was to investigate the transition phase in children's learning of symbolic representational skills in mathematics. This transitional phase takes place as they progress from the informal strategies that they learned independently of formal school instruction to the more formal school-learned skills of writing symbolic sentences to represent verbal problems and then solving them. The content was limited to addition and substraction problems. The project used the microcomputer to establish a direct link between writing symbolic number sentences and children's informal modeling processes. The researchers built this study upon their findings of previous studies.

For this study the researchers developed a computer program which enabled children to use two-dimensional simulations on a microcomputer display in place of actual physical objects to solve word problems. The children could produce sets of objects one at a time and could make a single set, two sets or remove elements from a set they had constructed.

A teaching experiment was carried out after the development of the computer program in order to: (1) validate the physical and conceptual features of the microcomputer program; (2) study the development of children's number-sentence writing ability; (3) develop and validate procedures of instruction related to use of the computer program; and (4) evaluate the effectiveness of the computer program and the related instructional procedures in linking the informal solution strategies of young children

and the formal symbolism of mathematics.

The four subjects for this clinical study were selected from the two first grade classes in a parochial school located in a middle-class neighborhood in Madison, Wisconsin. Individual interviews were conducted in order to observe each child's ability to express himself/herself and give clear explanations of the procedures and actual processes used to solve a given problem. The four selected subjects were taught a series of lessons individually by one of the two experimenters in the presence of a second adult observer. Following the instruction, each of the four subjects was given a brief individually administered problem solving interview.

In summary, three of the four subjects in the experiment consistently wrote appropriate number sentences and solved the problems using strategies which were consistent with their number sentences. None of the four subjects wrote correct sentences in the screening tasks given to them before the experiment. Three of the four subjects wrote correct number sentences for the compare problems and solved the problems using a strategy which modeled their number sentence rather than the structure of the problem. The same three children wrote correct noncanonical open sentences for the missing addend problem and they were generally successful in writing correct sentences for the missing minuend problem, on which almost no instruction was given.

The results strongly supported the conclusion that an instructional program based on the principles of this study

would be effective in teaching representational and formal problem solving skills for solving addition and subtraction word problems. Before instruction took place, the four subjects wrote inappropriate number sentences for all but the most straightforward addition and subtraction sentences. Prior to instruction the subjects ignored the number sentences. After instruction took place the four subjects could write number sentences to represent problems. They used this ability to solve a variety of problems using the computer. Three of the four subjects could apply this ability to solve problems without the computer.

The results of this study supported the suggestion that initial instruction in addition and subtraction should include noncanonical sentences. During and after instruction the four subjects consistently wrote noncanonical sentences to represent missing addend word problems. Pupil performance during the lessons also supported the conclusion that the instruction was successful in developing representational skills. The children were helped to understand the relationship between their own informal strategies and the formal mathematical representations. The four children's abilities to interact with the computer were very positive. No difficulties whatsoever were recorded. The computer seemed to allow children to rely upon their informal mathematics in an area of formal mathematics such as sentence writing. This experiment demonstrated that the computer may allow children to represent problems in a formal way without having completely learned the formal algorithms and number facts. These findings

suggested that instruction could be modified to make more effective use of children's natural ability to solve verbal problems in learning the formal mathematics of addition and subtraction.

Comments, Criticism, Questions and Concerns

The study seemed to be thoroughly planned, designed and completed. However, the total number of subjects was only four. These four students were carefully selected for the study from two first-grade classes in a parochial school located in a middle class area of Madison, Wisconsin; a city which also provides a home for a very large university. How much weight can be assigned to findings based on the observed performance of only four children in a parochial school serving a middle class population? These four children seemed to have regular access to computers during this study. Most children in ordinary school settings have little or no access to computers. Four subjects might be too few even for a pilot study.

Most readers probably agree with the researchers' suggestion that investigations concerning children learning mathematics and their use of computers when doing so should be continued. Perhaps a larger number of children from a variety of settings and backgrounds should be included in such future investigations.

How much of the children's improvement in performance could be attributed to the increased conceptualization of the mathematical processes involved and how much of this increased conceptualization could be attributed to the Piagetian-type two-dimensional representations of the word problems appearing

on the computer displays?

PUPIL GENERATED DIAGRAMS AS A STRATEGY FOR
SOLVING WORD PROBLEMS IN ELEMENTARY MATHEMATICS
Anna Vance Yancey (1981)

This study included a discussion of prior research that was unsurpassed in quality. The landmark studies in word problems dated back to 1935 and included the work of Suydam and Weaver, Pace, Shoecraft, Hansen and Brueckner.

This study was intended to test these six general hypotheses:

1. Children who are taught to produce their own diagrams of the inherent structure of word problems in elementary mathematics will perform better on tests of word problem solving ability than pupils who are provided an equivalent amount of instruction in other problem solving strategies.
2. The diagrams will be exhibited by the subjects in the experimental groups (Method A) on both a posttest and a delayed retention test.
3. The pupils producing the most correct diagrams on a test will achieve a higher score on that test.
4. Pupils taught the diagramming strategy shall exhibit

a more positive attitude toward mathematics than pupils taught by traditional methods.

5. Less than ten hours of instruction shall be required to achieve the above effects.
6. Those instructors who are unfamiliar with teaching pupils to generate their own diagrams shall demonstrate an ability to successfully use this strategy.

The 92 subjects participating in this study were members of four fourth grade classes at Centerfield Elementary School, Oldham Country School District, Kentucky. These students completed an attitude pretest, a pretest on solving word problems, six weeks of instruction on solving word problems (three 25 minute periods per week), a posttest on solving word problems, an attitude posttest and a retention test on solving word problems given six weeks after the posttest. Two of the groups received instruction based on a strategy of teaching the students to generate their own diagrams to represent the inherent structure of the word problems. This strategy of teaching was named Method A. The other two groups of students were taught with a teaching strategy based upon an eclectic approach to solving word problems. This was known as Method B. Each of the two investigators taught one group with Method A and another group with Method B.

The major results indicated that Method A students did learn to generate their own diagrams which represented the inherent structure of word problems. They demonstrated this new skill on the posttest and the retention test. Both groups,

A and B, showed statistically significant improvement from the pretest to posttest and retention test. The reference group did not. Although both groups, A and B, showed significant improvement from pretest to posttest, those students who were taught to generate their own diagrams (Method A) performed significantly better than the Method B students. Method A students also expressed more positive attitudes toward mathematics than did Method B students.

Within the Method A group there was a significant correlation between the number of correct student generated diagrams on the posttest and the retention test, as contrasted with Method B students generating an average or below average number of correct diagrams.

This study demonstrated the effects of teaching fourth grade students to generate their own diagrams representing the inherent structure of word problems. Teachers previously unfamiliar with the technique accomplished this in less than eight hours of pupil instruction.

The implications for this study include: (1) children can be taught to generate their own diagrams as a strategy in solving word problems; (2) performance on word problem tests improves after acquiring this strategy; and (3) the study of children's use of diagrams to solve word problems might be a useful instrument for examining pupil's mental processes.

The pedagogical implications are clear: (1) teachers should be taught to solve mathematics word problems by drawing

diagrams and (2) teachers should be encouraged to communicate this problem solving strategy to their students.

Comments, Criticism, Questions and Concerns

The review of the literature on word problem solving was outstanding. The six hypotheses for this study were clearly stated and easily located. This study included a definition of terms according to usage in this particular study. A list of ten questions for further research were offered by the investigator. These questions were included in the suggestions and questions for further research section of this document. The researchers did a thorough job of research and in reporting the results. The study was a worthy contribution to children and learning.

Some of the fourth graders in this study were on the threshold of their formal thinking stage. The technique of developing children's conceptual imagery through drawing seems like an ideal bridge between the concrete operational and formal stages as described in the work of Piaget.

THE EFFECT OF A MODEL DESIGNED TO FACILITATE MATHEMATICAL STORY PROBLEM-SOLVING SKILLS IN CHILDREN IN THE INTERMEDIATE ELEMENTARY GRADES

Florence C. Wilson (1982)

The purpose of this study was to develop a model instructional unit for teaching story problem-solving skills to fourth and fifth grade students and to measure the model's effectiveness.

Two statements of hypotheses were proposed for the study:

1. Fourth grade pupils taught with the new instructional model will demonstrate an increased ability to solve mathematics story problems when compared with a group of fourth grade elementary pupils taught with the conventional methods of instruction as measured by alternate forms of a pre-norm and post-norm referenced standardized test.
5. Fifth grade elementary school pupils taught with the new model instructional unit will demonstrate increased ability to solve mathematics word problems when compared with a group of fifth grade elementary pupils taught with the conventional methods of instruction as measured by alternate forms of a referenced standardized test.

The study included two null-hypothesis:

1. There will be no significant difference between samples of fourth grade elementary pupils taught with the new model instructional unit and the fourth grade students taught with the conventional methods of instruction.
2. There will be no significant difference between samples of fifth grade elementary pupils taught with the new model instructional unit and the fifth grade students exposed to the conventional methods of instruction.

The control group consisted of eight boys and six girls in the fourth grade and seven boys and five girls in the fifth grade for a total of 26 students.

The experimental group consisted of eight boys and seven girls in the fourth grade and six boys and six girls in the fifth grade for a total of 27 students.

The total number of participants in this study were 53 learners whose scores were below the 40th percentile in the CAT-Math. They were within the normal range of intelligence and had no learning disabilities.

The independent variables were the new model instructional unit and the usual textbook instruction.

The dependent variables were the posttest scores of the pupils adjusted for the disparity between the groups on the pretest and analyzed to determine the effects of the independent variables.

Levels nine and ten of forms seven and eight of the Iowa Tests of Basic Skills-Mathematics Problem Solving were administered as the pretest and posttest for both groups.

The results indicated that the model instructional unit was an apparently effective means of teaching mathematics story problem solving skills to the experimental group. This conclusion was qualified by the condition that this instructional unit of instruction included group-involved discussion illustration and demonstration of solving problems, i.e., the instructional unit alone could not account for the differential results.

The evidence indicates that the students who were taught

with the model instructional unit and the group techniques and demonstrations were able to solve more mathematics word problems than were solved by the students taught by the basic textbook methods of instruction as tested by Level nine or ten of Form eight of the Iowa Tests of Basic Skills-Mathematical Problem Solving.

Recommendations by the researcher for future researchers who wish to expand on this study include the following:

1. Field studies involving more teachers and a variety of larger population samples.
2. Results could be evaluated by grade and sex.
3. Level of intelligence should serve as a second covariate.
4. Populations should represent the full range of student achievement levels.
5. The instructional unit should be taught over a full year instead of a semester.
6. Teachers should be instructed in how to implement the unit before teaching it.

Comments, Criticism, Questions and Concerns

This study was thorough and competently reported. The review of the literature was nothing less than outstanding. The hypotheses were clearly stated and easily located. The design was appropriate. The summary, conclusions and recommendations were carefully presented. Perhaps the weakest aspect of the study was the almost total absence of information

about the model instructional unit. The testing of this unit was the topic of the entire study. The appendix contained only a three page descriptive overview of the model unit without any sample demonstrations or problems. The text of the study itself seemed to lack completely any description of the model unit.

In the conclusions the researcher attributed the higher test scores of the experimental group to the model instructional unit and a condition which was separate and apart from the model, i.e., group involved discussion, illustrations, and demonstrations of solutions to problems. Did the control group experience this 'extra' condition? Why wasn't this 'extra' condition treated as a part of the model unit or as a separate variable? Was it described in the design? Could it have been built-into the model unit?

According to the researcher, the subjects were randomly assigned to either the control or experimental groups. However, the random assignment was not effective. How can the random assignment of subjects be made more effective in future expanded studies?

The researcher suggested that future expanded studies take place over a one year period rather than a semester. The subjects in this study were fourth and fifth graders and some of them, especially the fifth graders, would be on the threshold of their formal thinking stage. Over a one year period many of the subjects could be crossing over into the psychological maturity which includes formal thinking abilities. When so, how much of the students' success could be attributed to their

new psychological-developmental maturity rather than to any particular instructional model?

The subjects in this study were all of normal intelligence. Was this characteristic typical of all school situations? Would it be true in randomly selected classes in other locations?

One of the researcher's most profound statements was not directly related to the experimental research itself. This important statement was that few textbooks or instructional programs meet the necessary criteria for effectively teaching problem solving to children.

One of the most profound references included by the researcher might have been Brownell (1942) who suggested that:

> A problem is not necessarily 'solved' just because the correct response* has been made. A problem is not truly solved unless the learner understands what he/she has done and knows why his/her actions were appropriate.

Florence Wilson's contribution seems compatible with the works of Piaget and Brownell.

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*A correct conditioned response?

SUMMARY

The experimental research of the 1980's seemed to focus on the children, instruction, and the word problem itself. The studies which focused on the children were concerned with these five major topics: (1) the children's own inventive processes for solving word problems; (2) conceptual understanding; (3) cognitive functioning; (4) difficulties with two-step problems; and (5) concept representation by using diagrams and drawings. Although one or two studies analyzed three-dimensional manipulatives for conceptual representation, there seemed to be a shortage of such studies.

The studies which focused on instruction were concerned with these eight major topics: (1) instructional models; (2) explicit instructions and explanations; (3) direct sequenced instruction; (4) small cooperative groups with some peer group instruction; (5) estimation as a method for pupils to preview a solution process; (6) the requirement that children restate the problem in their own words as a method of encouraging correct interpretation; (7) computer assisted instruction for teaching and reinforcing problem solving skills; (8) computer models for use in word problem solving research. There seemed to be a shortage of experimental studies which included the use of computers.

The studies which focused on the problem itself were

concerned with: (1) language; (2) readability; (3) structure; (4) format; and (5) syntax.

Experimental studies which focused on rating textbook series for effectiveness in preparing children for solving word problems seemed to be non-existent. Rating scales for measuring this kind of textbook effectiveness seem to be rare.

These studies seemed to indirectly indicate that the principles of developmental psychology need to be respected and applied even when teaching for specific behaviors and outcomes. More studies seem to be needed which might reveal the respective and combined values of developmental and behavioral psychologies for more effective children's word problem solving abilities. The emerging concern seems to be not that children know merely how to achieve a solution to a word problem but that they understand why the solution is correct, i.e., conceptualization should accompany an instructed process.

The criticisms of these experimental studies in terms of the way they were reported might include a concern for the very small samples and their effect on the credibility of the findings. Some of the studies lacked adequate definitions for the major terms used.

The studies reviewed in this document were selected because they seemed to be experimental or scientific in nature. Some of the reports for the studies did not seem to include statements of any problem, hypotheses or questions to be investigated. Regardless of adherence to any particular manual of style for reporting research, a clear and precise statement

of the hypothesis should be included in the description of the experiment. Perhaps the hypotheses to be tested should be italicized or underlined.

By whose authority can a study of any kind be called scientific or experimental and be afforded the rights, privileges and credibility thereof without a hypothesis, statement of the problem or at least a question to be answered?

MAJOR FINDINGS AND CONCLUSIONS LISTED

1. Teachers who provided explicit explanations and interacted meaningfully with students were more effective in helping them become successful problem solvers (Herrmann, 1986).

2. Children at the same conceptual level seem to differ in their ability to construct and maintain representations of sensory motor actions (Cobb, 1986).

3. There are some children who are able to reason about quantitive problems, i.e., they know the basic procedures of a given operation but may not use algorithmic procedures to find answers to verbal

problems. They may rely on direct modeling and counting (Romberg, 1983).

4. Early didactic models in a child's schooled learning experiences might exert considerable influence on problem solving behavior even after the acquisition of formal mathematical thought (Fischbein, 1985).

5. Four skill areas represented an immediate need for a sizeable number of sixth graders, i.e., computation, interpretation, reading, and the integration of the three previous skills for finding a solution for a word problem (Ballew and Hunter, 1983).

6. One possible effect of some kinds of instruction on problem solving behavior might be a shift from a variety of strategies to solve a variety of problems to a single strategy (Carpenter, 1981).

7. Asking the child to restate the problem correctly in his own words can help the teacher identify his/her representations of the problem. It might also help the child better understand the problem (Quintero, 1983).

8. Teachers can be more sensitive to the sequence of instruction when they understand the prerequisite

knowledge structures for solving certain problems. Different strategies can be adopted when teaching at different levels (Nesher, 1982).

9. In regards to word problems, teachers should help students to develop the ability to break down complex-compound sentences into several simple sentences (Wheeler and McNutt, 1983).

10. Students working in cooperative conditions outperformed students in individualistic competitive conditions when solving word problems (Johnson and others, 1980).

11. The use of drawings to organize the data in word problems was most helpful to students scoring low on cognitive ability tests (Threadgill-Sowder and others, 1985).

12. The use of diagrams, the appropriate reordering of number sequences, and the removal of extraneous information can improve the success rate in solving word problems (Cohen, 1981).

13. The problem structure and the overall pattern of relations between the quantities in the problem had an effect on problem solving success (Shalin and others, 1985).

14. Field-independent subjects solved more word problems correctly than did field-dependent subjects (Vaidya and others, 1981).

15. Most of the children had more difficulties with two-step word problems than one-step problems. These difficulties were conceptual and strategic. Children tended to use the same strategies for solving two-step word problems as they did for one-step problems, i.e., with a single strategy. Students improved their performance in the two-step word problems after having been told that the problems required a two-step solution (Quintero, 1984).

16. Instruction had a bearing on any changes in a child's invented behavior for solving word problems. Instruction had a major effect on the range and application of learned strategies (Moser and others, 1982).

17. When working in small groups, a solution to a word problem was available within three acts of the reading of the problem. In each group, one student contributed very little of the total answers and another student gave over half the answers (Guilbert and Leitz, 1982).

18. Presenting word problems by way of drawings was clearly

more effective than the standard words-only presentation (Threadgill-Sowder, 1982).

19. The small group working without teacher intervention seems to be a powerful learning tool (Noddings, 1982).

20. Children trained to solve basic types of word problems in a sequence achieved higher posttest scores than those children who were trained to solve an unsequenced arrangement of the same problems (Jones and others, 1985).

21. Reading ability played a major role in the solution of word problems and extraneous information reduced the accuracy of students' answers and increased the length of their test-taking times (Muth, 1982).

22. The direct instruction approach which included the teaching of prerequisite skills in a sequential manner and explicit teaching of problem-solving skills was more successful than a traditional approach. Time on task alone did not increase performance (Darch and others, 1983).

23. The practice of applying readability formulas to determine the grade level appropriateness of word problems was questioned. Readability scores a few

grades above or below grade level did not substantially affect students' abilities to solve word problems (Paul, 1986).

24. Most first graders were limited to direct symbolic representations of word problems (Carpenter and others, 1985).

25. Sixth graders rarely applied systematic problem analysis when confronted with a word problem. Their difficulties with word problems seemed to be caused by a lack of the attitudes and skills necessary to the act of analyzing a problem before attempting to compute a solution (DeCorte and others, 1981).

26. Problem solving skill might be acquired from sources other than formal schooling. This might be especially true in other cultures (Adetula, 1985).

27. There was a high correlation between the problems solved and those restated by the children correctly in their own words. A major source of the children's difficulty was in repeating the intensive quantity. The representation of the intensive quantity was particularly important in solving these word problems (Quintera, 1981).

28. (a) Children relied less on verbal and material strategies and more on mental strategies as the school year progressed. (b) The value of data collected by human beings observing the behavior of other human beings might surpass that of computer models indefinitely (DeCorte and others, 1985).

29. The concept of ratio and two-step word problems were a source of difficulty for children (Schwartz/Quintero, 1981).

30. More effective use of children's natural ability to solve verbal problems might be possible with properly modified instruction (Moser and Carpenter, 1982).

31. Instruction increased pupils' success in writing number sentences (Moser and Carpenter, 1982).

32. Computers might allow children to represent problems in a formal way without the learners having completely mastered tne formal algorithms and number facts (Moser and Carpenter, 1982).

33. Students who generated their own diagrams to represent word problems were more successful in solving them than students who did not (Yancy, 1981).

34. A model instructional unit was an apparently effective means of teaching mathematics story problem solving skills when it was accompanied by group involved discussions, illustrations, and demonstrations of word problem solutions (Wilson, 1982).

SUGGESTIONS AND QUESTIONS FOR FURTHER RESEARCH IN MATHEMATICS WORD PROBLEM SOLVING*

1. A focus on the qualitative dimensions of instruction provided by teacher educators should include studies which: (a) describe characteristics of the verbal interaction patterns of students who actively interpret instructional information; and (b) teacher educators who actively interpret the student responses to instruction.

2. Does the study of the logo computer language enable the student to solve word problems more effectively?

3. Are some step-by-step problem solving strategies more effective than others? For whom? Why?

*Please note that most of these suggestions and questions were the reviewer's own ideas. They were a result of his reading and analyzing each study. Some of the suggestions were explicitly stated by the researchers and an effort was made to identify those items for the reader.

4. Do teachers apply the work of Piaget when explaining and giving directions for word problems, i.e., do teachers include considerations for psychological, developmental readiness as well as mathematical readiness?

5. What can teacher educators do to encourage teachers to apply the work of Piaget when explaining and giving directions for word problems?

6. To what degree do repeated exercises in drill and practice cause mathematical mindsets which inhibit formal thinking and cause children to focus on single strategies for solving word problems?

7. How can we avoid the development of mathematical mind sets? Can we do so by efforts toward helping children develop an understanding of our number system?

8. Can the time at which a child enters the formal operations stage be affected by teaching? If so, how? If not, why not?

9. How much do instruction and instructions contribute toward molding children into convergent thinking patterns and the use of single strategies for solving a variety of problems? Is convergent thinking a

desirable outcome in the teaching of mathematics? How much does our concern for standardized test scores contribute toward the teaching for convergent thinking?

10. How often do teachers mistake a correct conditioned response for an internalized concept (conceptualization)?

11. How often do teachers teach concrete operational children (Piaget) as if they were formal thinkers? or How often do teachers respect both performance and developmental factors when teaching mathematics?

12. Should mathematics teachers also be teaching some language arts and reading? Should some reading and language arts teachers also be teaching some mathematics? How can the efforts of mathematics teachers and teachers of reading and language arts be combined for the purpose of improving student abilities in solving word problems?

13. Should the teaching of mathematics in the elementary school be multi-tracked? If so, should materials for a given grade level also be multi-tracked in difficulty including the syntax of word problems?

14. When should teachers supplement their own methods and strategies for teaching a given concept or skill with children's peer group instruction?

15. Are we moving into an era when conditions and tools for communicating demand that formal thought itself be redefined?

16. Should formal thought be redefined to include the cognitive restructuring of abstract concepts for slow and average learners?

17. Are we misassuming the existence of formal thought in some learners, i.e., in practice, do our teaching methods and materials assume that learners become formal thinkers simultaneously?

18. Should formal thought be redefined or cognitively restructured for unimpeded and gifted learners to include advanced formal thought, i.e., a computer language such as logo which is considered an advanced form of abstract thinking for some children? Indeed, logo is thought by some (Papert, 1980) to have the capacity to alter the problem solving process.

19. For how many days, weeks, months or years shall learners retain the ability to apply specific

mathematical operations for solving word problems when these operations were "behavioralistically" acquired without respect for the principles of developmental psychology?

20. Should teachers and publishers remove and edit out any structural format faults which interfere with student success in solving word problems?

21. Which specific difficulties that children have in solving word problems can be attributed to the way the problems are written?

22. Is there a correlation beween left or right brain hemisphere dominance and field dependence or independence?

23. Some studies indicate that successful mathematics students might be left-brain dominant. If so, are left-brain dominant students field dependent or field-independent?

24. Some studies indicate that field independent students are more successful in mathematics than field-dependent students. Should the development of a field independent cognitive learning style be encouraged in learners? If so, how should it be encouraged?

25. How might instruction (convergent thinking) increase children's effectiveness in solving word problems without inhibiting the development of their inventiveness (divergent thinking)?

26. If indeed, participation in small groups helps children become more effective problem solvers, is it because of the small group dynamics at work? The peer group instruction? Or a combination of both? Do only the students who are active participants in the group experience gains? Can gains be made in the problem solving skills by 'silent' participation in the group? How much does the emerging group 'pecking order' determine each member's success in that group? What are the factors which determine the 'pecking order' within the group, i.e., status and sociability, math and verbal ability, one's gender?

27. What are the long term effects on concept retention of rote, repetition, conditioning methods, and the meaningful applications of computational operations to manipulations of concrete objects and events? Is it possible to combine the best of these two schools of thought (behavioral and developmental psychologies)? If so how?

28. Should the teaching of reading be combined with computational skills as a method for developing

and assessing comprehension while simultaneously teaching and reinforcing problem solving skills? How much longer can the efforts of reading and mathematics teachers remain separated?

29. How much do traditional textbooks, curriculums and teaching strategies contribute toward student success in word problem solving? or How much do they contribute toward distracting from success, i.e., do they provide for the prerequisite sequenced skills which are needed for developing and bringing together the necessary conceptualizations, computational skills,and strategies for word problem solving?

30. The 'new mathematics' relied heavily on the contributions from mathematicians. Should the mathematics for the 1990's and the new century seek and consider the contributions of the developmental psychologists?

31. How much of word-problem solving is an act of creativity?

32. To what degree does investigation itself change the character or process of that which is being investigated (Harris, 1986)?

33. Do the cognitive processes of persons over 50 years

of age differ from those of younger persons? If so, then why? What effects did the absence of television in the lives of persons over 50 years of age have on their cognitive processes?

34. Erik DeCorte recommended the following hypothesis for further study: The acquisition by the learners of the heuristic estimation strategy leads to a qualitative improvement in their word problem solving, i.e., it is especially the heuristic estimation strategy which induces an improvement in learner's word problem solving and is the determining factor of the increase in that performance. In researching this hypothesis DeCorte recommended that special attention be given to the collection of qualitative data on pupils' problem solving processes before, during, and after the experimental teaching program.

35. Was it the estimation of the outcome itself that made a difference in DeCorte's learner's word problem solving abilities or was it the abbreviated 'passing through' the solution process, i.e., the 'dry-run' which was encouraged by the estimation? What other factors could be applied to encourage the 'dry-run' through a word problem?

36. Simple ratios and simple fractions seem to be highly

abstract concepts for children. Why? Do children have difficulties with the concepts of simple ratios and simple fractions for some of the same reasons they have difficulties with word problems?

37. In the minds of teachers, what is teaching for conceptualization?

38. Do teaching strategies for word problems which combine conceptual development with process result in increased retention as compared with teaching process alone?

39. re: Moser, James M., Carpenter, Thomas P. (1982). How much of the children's improvement in performance could be attributed to the increased conceptualization of the mathematical processes involved and how much of this increased conceptualization could be attributed to the Piagetian-type two-dimensional representations of the word problems appearing on the computer displays?

The following ten questions for further research were contributed by Anna Vance Yancy (1981) in her study titled: Pupil Generated Diagrams as a Strategy for Solving Word Problems in Elementary Mathematics:

40. Would the method (conceptual imagery through drawing) be of more or less benefit to pupils of more or less

aptitude than those subjects in this study?

41. Would the effect be more or less pronounced if compared to truly traditional instruction rather than the Method A/ Method B comparison used in this research?

42. Would the typical teacher adopt the drawing imagery techniques with those positive effects experienced by teachers in this study?

43. Would the technique be more or less effective at other grade levels?

44. Do the acquired pupil skills represent a permanent improvement in word problem solving ability?

45. Would the technique be as effective in other curriculum areas, eg., science?

46. Could the technique be successfully taught to students by workbooks which show pupils how to diagram the inherent structure of word problems?

47. Which type of learner would benefit most from which method of instruction?

48. Could student generated diagrams be used to diagnose

individual pupil difficulties with mathematical concepts?

49. Does the inability to diagram a problem indicate that a student does not understand the concepts involved, even if he calculates a correct answer?

50. How much do mathematics textbooks contribute or distract from children's success in word problem solving?

51. Have any rating scales been developed which recognize the presence or absence of specific strategies in mathematics textbooks which are known to contribute toward the encouragement of children's success in solving word problems?

52. Are curriculum concerns best served by Skinnerian principles?

53. Are children's needs best served by Piagetian concerns?

54. In the best interests of the efforts to improve children's problem solving abilities should the most meaningful contributions of both schools of thought, behavioral and developmental, be combined in order to maximize the effectiveness of these efforts? Should

these efforts be focused on teachers, children, textbooks and materials?

55. A possible paradox: Does the entire education effort unintentionally 'stack the deck' against word problem solving then laments the results and pleads for solutions; a victim of its own size and complexity which cannot always readily change itself?

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A P P E N D I X

THE THREE GENERAL SEMANTIC CATEGORIES OF ADDITION AND SUBTRACTION WORD PROBLEMS*

Name of The Category	Characteristics	Example
1. Combine	Involves a Static relationship between sets. Asking about the Union set or about one of two disjoint sub-sets.	There are 3 boys and 4 girls. How many children there are altogether?
2. Change	describes increase or decrease in some initial state to produce a final state.	John has 7 marbles. He lost 3 of them. How many marbles does John have now?
3. Compare	involves a static comparison between two sets. Asking about the difference set or about one of the sets where the difference set is given.	Tom has 6 marbles. Joe has 4 marbles. How many marbles does Tom have more than Joe?

*Nesher, P. and Others (1982).